

# WACHSTUMS- UND REGIONALÖKONOMIE

## EMPIRICAL EVIDENCE ON THE NEOCLASSICAL GROWTH MODEL

### S E M I N A R A R B E I T

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#### Abstract

This paper deals with the neoclassical Solow growth model and its empirical evidence in cross-country and regional panel data sets. It is shown that an extended version of the model, which accounts for human-capital accumulation, is qualitatively and quantitatively consistent with the observed differences in per capita income across the world.

vorgelegt von

C l a u d i u s R a f f l e n b e u l - S c h a u b

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- 
- Leiter des Seminars: Prof. Dr. Dalia Marin
  - Semester: Wintersemester 2000/01
  - Adresse des Verfassers: Rohmederstr. 19a, 80805 München
  - Telefon des Verfassers: 089-32423714
  - E-Mail des Verfassers: Rafflenbeul@gmx.de

T A B L E O F C O N T E N T S

	page
List of References	III
List of Figures and Tables	IV
I N T R O D U C T I O N	1
<u>I. THE NEOCLASSICAL GROWTH MODEL</u>	1
1. The Basic Solow Model	1
2. Concepts of Convergence	3
a) Absolute versus Conditional Convergence	4
b) Convergence and the Dispersion of Per Capita Income	5
<u>II. EMPIRICAL EVIDENCE ON THE MODEL</u>	6
1. Empirical Analysis of a Cross Section of Countries	6
a) Results for Growth Rates of GDP	7
2. Empirical Analysis of Regional Data	10
a) OECD Countries	10
b) U.S. States, Japanes Prefectures and European Regions	12
<u>III. EXTENSION OF THE SOLOW MODEL</u>	12
1. Adding Human-Capital Accumulation to the Model	13
C O N C L U D I N G O B S E R V A T I O N S	14
Appendix	16

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LIST OF FIGURES AND TABLES

		page
Figure 1	Convergence across countries: Growth rate versus initial level of real per capita GDP for 107 countries	7
Table 1	Simple regression for per capita growth	8
Figure 2	Growth rate from 1960 to 1985 versus 1960 real per capita GDP for 24 OECD countries	11
Table 2	Regression for OECD countries	11

## I N T R O D U C T I O N

The neoclassical growth model as promoted by Robert Solow<sup>1)</sup> and others during the 50s predicted that an economy's per capita growth is negatively related to its initial level of income per person. In other words, poor countries tend to grow faster in per capita terms than rich ones, assuming that all countries share the same exogenous parameters. Thus, homogenous economies converge over time.

In recent times, economists such as Barro [1991], Mankiw [1992] and Sala-I-Martin [1996] tested empirically the predictions of the neoclassical growth theory for validity and tried to estimate the speed of convergence across the economies of the world.

Their research is based mainly on a data set from Allan Summers and Robert Heston<sup>2)</sup> which provides internationally comparable levels on gross domestic product (GDP) for a large number of countries.

This paper gives a brief review of the neoclassical growth model with exogenous saving rates and summarizes the empirical evidence on the convergence hypothesis.

Section I starts with the basic Solow model and the different concepts of convergence. In Section II the evidence on the model is analysed using a sample of cross-country data. Finally, the Solow model is extended by adding human-capital accumulation in Section III.

### I. THE NEOCLASSICAL GROWTH MODEL

#### 1. The Basic Solow Model

The main elements of the basic Solow model are the exogenous and constant parameters saving rate, denoted  $s$ , population growth, denoted  $n$ , and labor-augmenting technological progress, denoted  $g$ , and a constant return to scale Cobb-Douglas production function. Output,  $Y$ , depends on two inputs, the capital stock,  $K$ , and the labor force,  $L$ , which are paid their marginal products. For all  $K > 0$  and  $L > 0$ , the

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<sup>1)</sup> See SOLOW, pp. 65-94.

<sup>2)</sup> See SUMMERS/HESTON, pp. 327-368.

production function exhibits positive and diminishing marginal products with respect to each input. It takes the form

$$Y(t) = F [K(t), L(t) \cdot A(t)], \quad (1)$$

or

$$Y(t) = K(t)^\alpha \cdot [A(t) \cdot L(t)]^{1-\alpha} \quad 0 < \alpha < 1, \quad (2)$$

where  $A > 0$  is the level of technology, also defined as efficiency of labor.  $L$  and  $A$  varies over time,  $t$ , because of the growth rates  $n$  and  $g$ . Therefore, the number of effective units of labor,  $A(t) \cdot L(t)$ , grows at the rate  $n + g$ .

Output is assumed to be a homogenous good that can be consumed,  $C(t)$ , or invested,  $I(t)$ , to create new units of physical capital,  $K(t)$ .  $s > 0$  is the fraction of output that is invested and  $K$  depreciates at the constant rate  $\delta > 0$ . Defining  $k = K/AL$  as the stock of physical capital per effective unit of labor and  $y = Y/AL$  as the level of output per effective unit of labor the intensive form of the production function is written as

$$y(t) = F [k(t), 1] = f [k(t)] = k(t)^\alpha. \quad (3)$$

The net increase in  $k$  over time equals gross investment less effective depreciation, which is the fundamental dynamic equation of the Solow model given by

$$\dot{k}(t) = s \cdot f [k(t)] - (n + g + \delta) \cdot k(t), \quad (4)$$

where a dot over a variable denotes differentiation with respect to time. This nonlinear equation depends only on  $k$ . In the steady state  $\dot{k} = 0$  and the corresponding value of  $k$  is denoted  $k^*$ , which satisfies the condition

$$s \cdot f (k^*) = (n + g + \delta) \cdot k^*. \quad (5)$$

The per efficiency unit quantities  $k$ ,  $y$  and  $c$  are constant in the steady state. Therefore, the per capita variables  $K/L$ ,  $Y/L$  and  $C/L$  grow in the steady state at the rate  $g$  and the

level variables  $K$ ,  $Y$ , and  $L$  grow at the rate  $n + g$ . The steady state represents the long-run equilibrium of the economy. The long-run growth rates in the model are determined entirely by exogenous elements. Once the economy is in the steady state, the growth rate of income per capita is independent of the saving rate and the level of the production function and it depends only on the rate of technological progress.

Transitional dynamics show how the economy's per capita income converges towards its long-run steady-state value.<sup>3)</sup> The growth rate, denoted  $\gamma_k$ , of  $k$  is given by

$$\gamma_k = \dot{k}/k = s \cdot f(k)/k - (n + g + \delta). \quad (6)$$

For any initial value,  $k(0) > 0$ , the economy converges to its unique steady-state capital-labor ratio  $k^* > 0$ . The farther away  $k(t)$  is from  $k^*$ , the higher is  $|\gamma_k|$ . The source behind this convergence result is the diminishing returns to reproducible capital, one of the main elements of the neoclassical production function: when  $k$  is relatively low, the average product of capital,  $f(k)/k$ , is relatively high. Thus, the growth rate,  $\gamma_k$ , is also relatively high. Equation (5) implies that the steady-state capital-labor ratio of a Cobb-Douglas production function is defined by

$$k^* = [s/(n + g + \delta)]^{1/(1-\alpha)}. \quad (7)$$

$k^*$  and also  $y^*$ , by substituting equation (7) in the production function, is related positively to  $s$ , and negatively to  $n$ ,  $g$  and  $\delta$ .

## 2. Concepts of Convergence

As shown above, in the Solow model smaller values of  $k$  are associated with larger values of  $\gamma_k$ . This conclusion leads to the important question about the presents of convergence across economies: Do economies that start off poor, with lower capital per person, subsequently grow faster than economies that start off rich? In other words, is the degree of income inequality across the world falling over time?

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<sup>3)</sup> See BARRO/SALA-I-MARTIN, pp. 22-24.

### a) Absolute versus Conditional Convergence

The hypothesis that poor economies tend to grow faster in per capita terms than rich ones is referred to as "absolute  $\beta$ -convergence".<sup>4)</sup> When comparing annual data on real per capita GDP for a group of economies we estimate the following regression

$$\log (y_{i, t+T}/y_{i, t})/T = a - \beta \cdot \log (y_{i, t}) + \varepsilon_{i, t}, \quad (8)$$

where  $\log (y_{i, t+T}/y_{i, t})/T$  is economy  $i$ 's annual growth rate of GDP,  $\log (y_{i, t})$  is its logarithm of GDP per capita at time  $t$  and  $\varepsilon_{i, t}$  is a disturbance term. The condition  $\beta > 0$ , exhibiting a negative relation between growth rate and GDP, implies absolute  $\beta$ -convergence in the data. Hence, a higher coefficient  $\beta$  corresponds to a higher convergence speed. The speed of convergence,  $\beta$ , is measured by a log-linearisation of the dynamic equation around the steady state:

$$\beta = (1 - \alpha) \cdot (n + g + \delta), \quad (9)$$

where  $\alpha$  is the capital share in the production function.<sup>5)</sup> Since  $0 < \alpha < 1$  in the Solow model, the prediction is that  $\beta > 0$ . Thus,  $\beta$ -convergence is existing. Obviously, the speed of convergence is not affected by the saving rate,  $s$ . To see the quantitative implications of the exogenous parameters in equation (9) the reasonable benchmark values  $n = 0.01$  per year (the rate of population growth in recent decades),  $g = 0.02$  per year (the rate of productivity growth equal to the long-term growth rate of real GDP) and  $\delta = 0.05$  per year (the rate of depreciation for the overall stock of structures and equipment) are assumed. For these given values of  $n$ ,  $g$  and  $\delta$  the convergence coefficient,  $\beta$ , is determined by the capital share,  $\alpha$ . The conventional value of  $\alpha$  estimated under the assumptions of neoclassical theory (perfect competition and no externalities) is about 0.3. If the capital share is 0.3, then equation (9) implies a relatively

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<sup>4)</sup> See SALA-I-MARTIN, p. 1020.

<sup>5)</sup> See BARRO/SALA-I-MARTIN, pp. 36-38.



high convergence speed of  $\beta = 0.056$  per year, which implies that it will take about 12.5 years to eliminate the half of the distance between the initial level of per capita income and the steady state level.<sup>6)</sup>

The Solow model, however, does not predict absolute  $\beta$ -convergence in all circumstances. One key assumption of the model is structural similarity of the economies that are compared. That is, the economies must share exactly the same parameters  $s$ ,  $n$ ,  $g$  and  $\delta$  and also have the same production function, and thus, the same steady state. If this strong assumption of homogeneity, that is not very likely to hold true in the real world, is relaxed the analysis has to be modified. Heterogenous economies converge to their very own steady-state positions only and the speed of this convergence is negatively related to the distance from their own steady-state values. This concept is referred to as "conditional  $\beta$ -convergence".<sup>7)</sup>

The two concepts of absolute and conditional  $\beta$ -convergence coincide only if the compared economies share the same steady state. If this is not the case, all the Solow model predicts is conditional  $\beta$ -convergence.

#### b) Convergence and the Dispersion of Per Capita Income

The concept of  $\beta$ -convergence discussed so far must be distinguished from an alternative concept of convergence, which is referred to as " $\sigma$ -convergence".<sup>8)</sup>  $\sigma$ -convergence means that the dispersion of real per capita income across a group of economies tends to decrease over time:

$$\sigma_{t+T} < \sigma_t, \quad (10)$$

where  $\sigma_t$  is the time  $t$  standard deviation of  $\log(y_{i,t})$  across all economies.

There is a close relation between this two different concepts of convergence. If the GDP levels across economies become more similar ( $\sigma$ -convergence) it must be the case that

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<sup>6)</sup> See BARRO/SALA-I-MARTIN, pp. 36-38.

<sup>7)</sup> See SALA-I-MARTIN, p. 1027.

<sup>8)</sup> See SALA-I-MARTIN, p. 1020.

poorer economies are growing faster (absolute  $\beta$ -convergence). In other words, it is impossible to overcome the income inequality across the economies without observing  $\beta$ -convergence. The existence of  $\beta$ -convergence is a necessary condition for the existence of  $\sigma$ -convergence. But, as mentioned above, the two convergence concepts are not identical.  $\beta$ -convergence could exist without the existence of  $\sigma$ -convergence. That is, it is possible that the initially poor economies are growing faster without observing the dispersion of per capita income across economies to decline. If the poor economies grow so much faster than the rich ones the formerly poor economies might overshoot and end up ahead of the formerly rich ones. Hence, the dispersion across the economies has not necessarily decreased, they just change their ranking.

The conclusion is that the existence of  $\beta$ -convergence is a necessary but not a sufficient condition for the existence of  $\sigma$ -convergence.  $\beta$ -convergence relates to the mobility of the different countries within the given distribution of income across the world, while  $\sigma$ -convergence is related to whether or not this distribution of income across the world declines over time. The following analysis deals with  $\beta$ -convergence only.

## II. EMPIRICAL EVIDENCE ON THE MODEL

The key property of the Solow growth model is the prediction of conditional  $\beta$ -convergence. Economies that start out proportionally below their own steady-state position tend to grow faster. Homogenous economies, which share the same structural parameters and technologies, converge to the same steady state. Only then, absolute  $\beta$ -convergence applies and poor economies tend to grow faster than rich ones. In this section, these convergence predictions are tested by looking at the convergence behavior of different countries and regions.

### 1. Empirical Analysis of a Cross Section of Countries

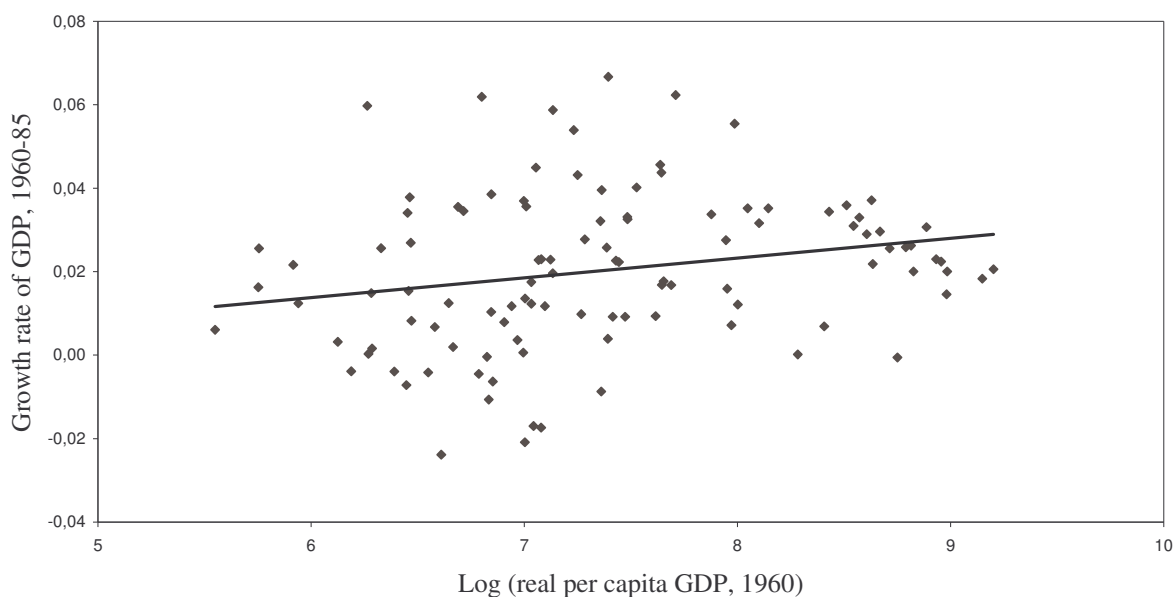
First, the development of a large set of countries across the world is examined. The data set of internationally comparable GDP levels for more than hundred countries, starting by the year 1960, is taken from the Summers-Heston "Penn World Tables 5.6" (PWT)

which can be found, for example, in the internet under the address <http://cansim.epas.utoronto.ca/pwt/> (at the University of Toronto).

#### a) Results for Growth Rates of GDP

Figure 1 plots the average annual growth rate of real per capita GDP between 1960 and 1985 against the log of real GDP per capita (in 1985 international prices) at the start of the period, 1960, for 107 countries.<sup>9)</sup> The figure shows that this sample rejects the hypothesis of absolute  $\beta$ -convergence across the world because the relation between the growth rate and the initial level of GDP is not negative. In fact, the correlation is 0.224 (Pearson correlation coefficient) and the fit is far from impressive.

Figure 1. Convergence across countries: Growth rate versus initial level of real per capita GDP for 107 countries.



The regression for annual average growth rates of per capita real GDP from 1960 to 1985 is shown in table 1.

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<sup>9)</sup> For a list of the countries see Appendix.

Table 1  
Simple regression for per capita growth

Dependent variable: Growth rate GDP per capita 1960-85	
Estimation method	OLS*
Constant	-0.0146 (0.015)
log (GDP 1960)	-0.00473 (0.0020)
adjusted R <sup>2</sup>	0.0410
standard error of regression	0.0182
N	107

\* The regressions uses ordinary least squares to estimate the equation

$$\gamma_{i,t,t+T} = a - \beta \cdot \ln (y_{i,t}) + \varepsilon_{i,t,t+T}$$

(in parentheses: standard errors)

The estimated speed of absolute  $\beta$ -convergence across the countries is negative,  $\beta = -0.00473$  (standard error = 0.002), so the relation between growth rate and initial level of GDP is positive. The adjusted R<sup>2</sup>, 0.041, is also poor. Thus, during the period of 25 years, poor countries did not grow faster than rich ones. The 107 countries did not converge in the sense of absolute  $\beta$ -convergence. Since the Solow growth model only predicts conditional  $\beta$ -convergence, the absence of absolute  $\beta$ -convergence says little about the validity of the model in the real world. To reconcile the model with the data the existence of conditional  $\beta$ -convergence has to be proven.

To test the hypothesis of conditional  $\beta$ -convergence the steady state of each country has to be held constant, somehow. There are two ways to approach this problem. Either some structural variables that distinguish the countries could be introduced into the regression to proxy for the steady state or the analysis could be restricted to a subset of countries for which the assumption of similar steady states is reasonable. Trying the first way additional variables enter into the estimation equation (8) in the form of

$$\gamma_{i,t,t+T} = a - \beta \cdot \log (y_{i,t}) + \psi \cdot X_{i,t} + \varepsilon_{i,t,t+T}, \quad (11)$$

where  $X_{i,t}$  is a vector of the variables that hold constant the steady state. If the estimate of  $\beta$  is positive once  $X_{i,t}$  is held constant, the data exhibits conditional  $\beta$ -convergence. The question remains, however, which variables are appropriate for successfully

holding constant the steady state. In theory, the Solow model predicts that the steady state is related positively to the saving rate and the level of the production function and negatively to population growth, technological progress and depreciation. Barro [1991] tested a large set of various variables such as different school-enrollment rates, literacy rates, fertility and mortality rates, different investment ratios, government expenditures, proxies for political instability, economic systems and market distortions, and dummies for subsets of countries.<sup>10)</sup> Some of the variables are significant and display a strong influence on the GDP growth rates. For example, the 1960 values of school-enrollment rates at the secondary and primary levels. These variables are based on data from the United Nations and measure the number of pupils enrolled in designated grade levels relative to the total population of the corresponding age group. Barro shows that the partial correlation between per capita growth and the school-enrollment variables is 0.73 and that increases in initial GDP per capita with the school-enrollment variables held fixed are strongly negatively related to subsequent growth.<sup>11)</sup> Moreover, given the initial level of per capita GDP, the growth rate is substantially positively related to the school-enrollment variables. This result highlights the importance of initial human capital, which the school-enrollment variables proxy for, for the differences of the growth rates across countries. Poor countries catch up with rich ones only if the poor countries have high human capital in relation to their low level of per capita income. The data is, in this modified sense, consistent with the conditional  $\beta$ -convergence hypothesis of the Solow model. Another variable significantly positively related to GDP growth rates is the average share of real investment in real GDP (a measure for the saving rate) as predicted in the Solow model. Significantly negatively related to GDP growth rates are the fertility rate (a measure for population growth), the ratio of government consumption expenditure to GDP, some variables of political instability, a proxy for price distortions and some regional dummy variables.<sup>12)</sup> These effects are consistent with neoclassical growth theory. All the variables together seem to be a good proxy for the steady state and when they are held constant in a regression the estimate of  $\beta$  becomes significantly positive. In other words, the partial correlation between per capita growth rates and the

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<sup>10)</sup> See BARRO, pp. 407-443.

<sup>11)</sup> See BARRO, pp. 416-417.

<sup>12)</sup> Also see BARRO/SALA-I-MARTIN, pp. 414-461.

initial level of per capita GDP becomes substantially negative, as predicted by the Solow model. Sala-I-Martin [1996] founded an estimated speed of conditional convergence across the world close to 2 percent per year.<sup>13)</sup>

The conclusion is that the empirical cross-country analysis supports the conditional  $\beta$ -convergence hypothesis of the neoclassical growth model qualitatively. The differences in per capita growth rates across countries are related systematically to a set of quantifiable explanatory steady-state variables. There is an evidence of diminishing growth rates of income as economies approach their long-run steady states.

## 2. Empirical Analysis of Regional Data

The second way to test the convergence hypothesis of the Solow model by holding constant the steady state is to restrict the analysis to a subset of countries, or regions within countries, for which the assumption of similar steady states is reasonable. If this assumption is true the Solow model predicts even absolute  $\beta$ -convergence. That is, poor economies tend to grow faster than rich ones.

### a) OECD Countries

One relatively homogenous subset of the world-wide sample are the industrial countries of the Organisation for Economic Co-operation and Development (OECD). Although differences in preferences and institutions might exist between the OECD countries, these differences are likely to be small and the assumption of a similar steady state is not erroneous. Figure 2 plots the average annual growth rate of real per capita GDP between 1960 and 1985 against the log of real GDP per capita at the start of the period, 1960, for the 24 countries that became OECD members before 1985. The figure shows a downward-sloping regression line. Thus, the OECD countries exhibit absolute  $\beta$ -convergence. The Pearson correlation coefficient is 0.672.

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<sup>13)</sup> See SALA-I-MARTIN, p. 1024.

Figure 2. Growth rate from 1960 to 1985 versus 1960 real per capita GDP for 24 OECD countries.

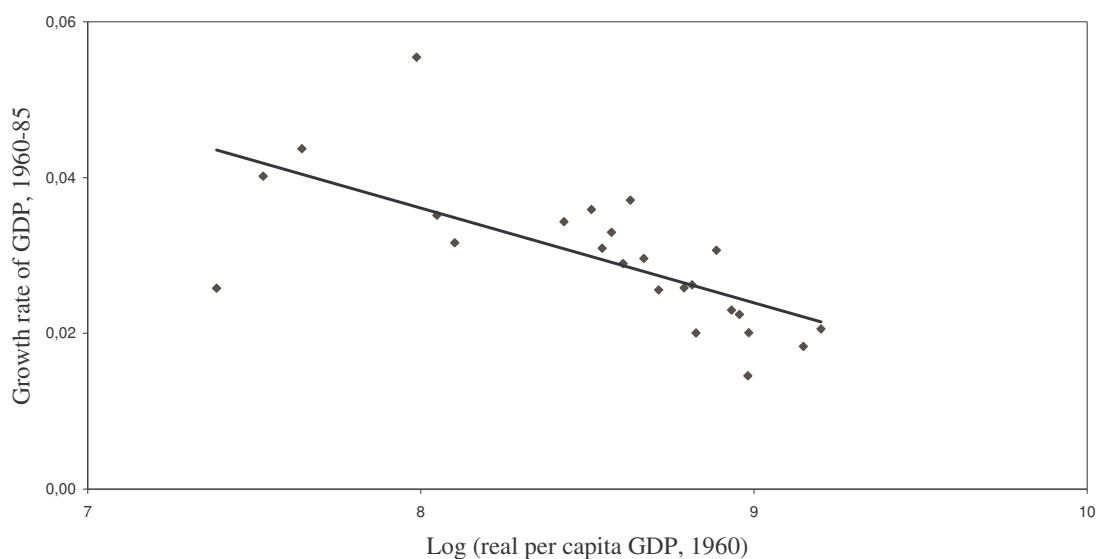


Table 2 shows the regression for annual average growth rates of per capita real GDP from 1960 to 1985.

Table 2  
Regression for OECD countries

Dependent variable: Growth rate GDP per capita 1960-85	
Estimation method	OLS*
Constant	0.1330 (0.024)
log (GDP 1960)	0.0122 (0.003)
adjusted R <sup>2</sup>	0.4270
standard error of regression	0.0069
N	24

\* The regressions uses ordinary least squares to estimate the equation

$$\gamma_{i,t,t+T} = a - \beta \cdot \ln(y_{i,t}) + \varepsilon_{i,t,t+T}$$

(in parentheses: standard errors)

The estimated speed of absolute  $\beta$ -convergence across the OECD countries is positive,  $\beta = 0.0122$  (standard error = 0.003), so the relation between growth rate and initial level of GDP is significantly negative. The adjusted R<sup>2</sup> is 0.427. When the additional

conditioning variables, which were used in the world-wide sample, are introduced into the OECD regression and held constant, the values of the estimated  $\beta$  and the adjusted  $R^2$  are even better. The estimated speed of convergence across the OECD countries is close to 2 percent per year (similar to the conditional convergence of the world-wide sample).<sup>14)</sup> It is evident that the convergence of per capita GDP levels across the OECD countries supports the convergence hypothesis of the neoclassical growth model qualitatively.

#### b) U.S. States, Japanes Prefectures and European Regions

Barro and Sala-I-Martin [1995] also estimate the convergence behavior for per capita income across 48 U.S. states for a 110-year period between 1880 and 1990, across 47 Japanese prefectures for the period 1930 to 1990 and across 90 regions within eight European countries (Germany, the United Kingdom, France, Italy, Spain, Belgium, the Netherlands and Denmark) for the 40-years between 1950 and 1990.<sup>15)</sup> The results of the estimated  $\beta$  and the goodness of fit across the regional samples are similar to that of the OECD countries. Poor regions tend to grow faster in per capita terms than rich ones. All of the samples exhibit significantly regional convergence and estimate a speed of  $\beta$ -convergence around 2 percent per year.<sup>16)</sup> The samples display conditional  $\beta$ -convergence as well as absolute  $\beta$ -convergence because the convergence even applies when no explanatory variables other than the initial level of per capita income are held constant. These results are, again, consistent with the qualitative convergence predictions of the neoclassical growth model if the regions within a sample share the same steady-state position, as assumed.

### III. EXTENSION OF THE SOLOW MODEL

One surprising finding of the results so far is that the speed of convergence,  $\beta$ , across the various contexts has been estimated to be within the narrow range around 2 percent

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<sup>14)</sup> See SALA-I-MARTIN, p. 1024.

<sup>15)</sup> See BARRO/SALA-I-MARTIN, pp. 382-413.

<sup>16)</sup> Also see SALA-I-MARTIN, p. 1024.



per year. This slow speed of convergence implies that it will take about 35 years to eliminate the half of the distance between the initial level of per capita income and the steady-state level.<sup>17)</sup> That is a strong contradiction to the quantitative prediction of the Solow model in Section I,  $\beta = 0.056$ , if the capital share is 0.3. The estimate of  $\beta = 0.02$ , together with the benchmark values of the other structural parameters, rather implies that the capital share is  $\alpha = 0.75$ . The value of  $\alpha = 0.75$  suggests that the basic Solow model tends to overestimate the speed of conditional convergence and is, therefore, not completely successful. For the model to be quantitatively consistent with the empirical results, it needs to be augmented so that the relevant capital share is large enough.

### 1. Adding Human-Capital Accumulation to the Model

As stated in Section II, the school-enrollment variables, a proxy for human capital, are substantially positively related to growth rates in the cross-country evidence. In other words, human capital seems to be of high importance to the process of growth. Mankiw, Romer, Weil [1992] propose a broader understanding of capital in the Solow model that consists not only of physical capital but also of human capital.<sup>18)</sup> They extended the basic Solow model of the first section by including human-capital accumulation into the production function:

$$Y(t) = K(t)^\alpha \cdot H(t)^\lambda \cdot [A(t) \cdot L(t)]^{1-\alpha-\lambda} \quad 0 < \alpha + \lambda < 1, \quad (12)$$

where  $H$  is the stock of human capital and all other variables are defined as before. Income can be used on a one-for-one basis for consumption or investment in either type of capital. Human capital depreciates at the same rate as physical capital,  $\delta$ .  $s_k$  is the fraction of income invested in physical capital and  $s_h$  is the fraction invested in human capital. The fundamental dynamic equations are now given by

$$\dot{k}(t) = s_k \cdot y(t) - (n + g + \delta) \cdot k(t), \quad (13a)$$

<sup>17)</sup> See BARRO/SALA-I-MARTIN, pp. 36-38.

<sup>18)</sup> See MANKIW/ROMER/WEIL, pp. 407-437.

$$\dot{h}(t) = s_h \cdot y(t) - (n + g + \delta) \cdot h(t), \quad (13b)$$

where  $h = H/AL$  is the stock of human capital per effective unit of labor and all other variables are defined as before. The new steady-state capital-labor ratio is defined by

$$\begin{aligned} k^* &= [s_k^{1-\lambda} \cdot s_h^\lambda / (n + g + \delta)]^{1/(1-\alpha-\lambda)} \\ h^* &= [s_k^\alpha \cdot s_h^{1-\alpha} / (n + g + \delta)]^{1/(1-\alpha-\lambda)} \end{aligned} \quad (14)$$

and  $\beta$ , the speed of convergence, is now given by

$$\beta = (1 - \alpha - \lambda) \cdot (n + g + \delta). \quad (15)$$

If  $\alpha = 0.3$ ,  $n = 0.01$ ,  $g = 0.02$  and  $\delta = 0.05$  as assumed and  $\beta = 0.02$  as in the samples, then the share of human capital,  $\lambda$ , would equal 0.45. This is a reasonable value of  $\lambda$  in the real world.<sup>19)</sup> The extended model, with human capital, is now quantitatively consistent with the empirical results. The convergence speed of a production function like  $Y = K^{0.3} \cdot H^{0.45} \cdot (A \cdot L)^{0.25}$  is 2 percent per year as estimated.

## C O N C L U D I N G O B S E R V A T I O N S

This paper indicates the international empirical evidence on the Solow growth model. The extended Solow model, which includes human capital as well as physical capital, is qualitatively and quantitatively consistent with the cross-country data and provides a good framework to understand the differences in income per capita around the world. First, the cross-country sample displays that poor countries have not grown faster than rich ones in absolute terms. However, holding constant some explanatory variables like initial human capital (proxied by school-enrollment rates), the saving rate and population growth, that proxy for the different steady states of the countries, the world-wide sample exhibits conditional  $\beta$ -convergence. The estimated speed of conditional

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<sup>19)</sup> See MANKIW/ROMER/WEIL, p. 417.

convergence is close to 2 percent per year as predicted by the extended model. Second, the regional samples, for example of the 24 OECD countries, converge even in absolute sense. The estimated speed of convergence is, again, close to 2 percent per year. This predicted convergence speed of 2 percent per year is very small. It suggests that it will take about 35 years to eliminate the half of the distance between the initial level of per capita income and the steady state level.

However, the Solow model simplifies many aspects of the real world, or it omits them altogether. For example, the model does not explain why the exogenous variables like the saving rate and the technological progress vary so much from country to country. The understanding of economic growth is not yet completed. There are more sophisticated growth models needed which can explain how private decisions and public policy affect the variables taken exogenous in the Solow model.

A P P E N D I X

List of Countries

No.	Country	GDP per capita		Growth rate 1960-85 GDP
		1960	1985	
1	Algeria	1710	2988	2,2 %
2	Angola	928	711	-1,1 %
3	Benin	1092	1108	0,1 %
4	Botswana	525	2337	6,0 %
5	Burkina Faso	457	495	0,3 %
6	Burundi	631	527	-0,7 %
7	Cameroon	634	1487	3,4 %
8	Central African Republic	699	630	-0,4 %
9	Chad	743	409	-2,4 %
10	Congo	1107	2697	3,6 %
11	Egypt	804	1953	3,6 %
12	Ethiopia	257	299	0,6 %
13	Gabon	1779	4072	3,3 %
14	Ghana	886	792	-0,4 %
15	Ivory Coast	1101	1545	1,4 %
16	Kenya	646	794	0,8 %
17	Liberia	721	853	0,7 %
18	Madagascar	1187	769	-1,7 %
19	Malawi	380	518	1,2 %
20	Mali	528	532	0,0 %
21	Mauritania	785	824	0,2 %
22	Mauritius	2840	4226	1,6 %
23	Morocco	825	1956	3,5 %
24	Mozambique	1145	749	-1,7 %
25	Niger	537	559	0,2 %
26	Nigeria	560	1062	2,6 %
27	Rwanda	535	776	1,5 %
28	Senegal	1062	1163	0,4 %
29	Somalia	1100	653	-2,1 %
30	South Africa	2185	3322	1,7 %
31	Swaziland	1240	2198	2,3 %
32	Tanzania	315	473	1,6 %
33	Togo	371	637	2,2 %
34	Tunisia	1095	2758	3,7 %
35	Uganda	596	540	-0,4 %
36	Zaire	487	442	-0,4 %
37	Zambia	946	808	-0,6 %
38	Zimbabwe	998	1216	0,8 %

39	Bangladesh	939	1216	1,0 %
40	Burma	316	599	2,6 %
41	Hong Kong	2231	10599	6,2 %
42	India	769	1050	1,2 %
43	Iran	2987	4043	1,2 %
44	Israel	3447	8310	3,5 %
45	Japan*	2943	11771	5,5 %
46	Jordan	1158	3561	4,5 %
47	Korea, Rep.	898	4217	6,2 %
48	Malaysia	1409	4146	4,3 %
49	Nepal	637	936	1,5 %
50	Pakistan	644	1262	2,7 %
51	Philippines	1133	1542	1,2 %
52	Singapore	1626	8616	6,7 %
53	Sri Lanka	1253	2045	2,0 %
54	Syria	1577	4240	4,0 %
55	Taiwan	1255	5449	5,9 %
56	Thailand	940	2463	3,9 %
57	Austria*	5137	11131	3,1 %
58	Belgium*	5469	11285	2,9 %
59	Cyprus	2075	6486	4,6 %
60	Denmark*	6730	12969	2,6 %
61	Finland*	5283	12051	3,3 %
62	France*	5820	12206	3,0 %
63	Germany, West*	6569	12535	2,6 %
64	Greece*	2086	6224	4,4 %
65	Iceland*	4974	12209	3,6 %
66	Ireland*	3299	7275	3,2 %
67	Italy*	4580	10808	3,4 %
68	Luxembourg*	7977	13175	2,0 %
69	Malta	1383	5321	5,4 %
70	Netherlands*	6087	11539	2,6 %
71	Norway*	5592	14144	3,7 %
72	Portugal*	1857	5070	4,0 %
73	Spain*	3128	7536	3,5 %
74	Sweden*	7573	13451	2,3 %
75	Switzerland*	9399	14864	1,8 %
76	Turkey*	1615	3077	2,6 %
77	United Kingdom*	6808	11237	2,0 %
78	Barbados	2637	6131	3,4 %
79	Canada*	7240	15589	3,1 %
80	Costa Rica	2090	3184	1,7 %
81	Dominican Rep.	1188	2111	2,3 %
82	El Salvador	1433	1831	1,0 %

83	Guatemala	1661	2090	0,9 %
84	Haiti	921	911	0,0 %
85	Honduras	1034	1387	1,2 %
86	Jamaica	1761	2215	0,9 %
87	Mexico	2825	5621	2,8 %
88	Nicaragua	1623	1790	0,4 %
89	Panama	1568	3499	3,2 %
90	Trinidad and Tobago	5623	9701	2,2 %
91	United States*	9908	16570	2,1 %
92	Argentina	4481	5324	0,7 %
93	Bolivia	1133	1754	1,7 %
94	Brazil	1780	4017	3,3 %
95	Chile	2897	3467	0,7 %
96	Colombia	1686	2968	2,3 %
97	Ecuador	1457	2913	2,8 %
98	Guyana	1574	1265	-0,9 %
99	Paraguay	1172	2072	2,3 %
100	Peru	2031	2565	0,9 %
101	Uruguay	3955	3969	0,0 %
102	Venezuela	6313	6225	-0,1 %
103	Australia*	7754	13583	2,2 %
104	Fiji	2108	3281	1,8 %
105	Indonesia	641	1651	3,8 %
106	New Zealand*	7953	11443	1,5 %
107	Papua New Guinea	1208	1619	1,2 %

\* the 24 OECD members before 1985